# Estimating the Cost of Capital 

In addition to cash flow, firm value is also a function of the firm's cost of capital. This chapter covers how a private firm's cost of capital is calculated. The financial costs associated with financing assets is termed the cost of capital because it reflects what investors require in the form of expected returns before they are willing to commit funds. In return for funds committed, firms typically issue common equity, preferred equity, and debt. These components make up a firm's capital structure. Each of these components has a specific cost to the firm based on the state of the overall investment markets, the underlying riskiness of the firm, and the various features of each capital component. For example, a preferred stock that is convertible into common stock has a different capital cost than a preferred stock that does not have a conversion feature. Common stocks that carry voting rights have a lower cost of capital than common stocks that do not. This difference occurs because the common stock with voting rights is more valuable, and hence the return required on it is necessarily lower than the same common stock without voting rights.

A typical public firm has a capital structure that includes common equity and debt and, to a lesser extent, preferred stock. This contrasts to private firms, which generally have common stock and debt. S corporations, which represent the tax status of a significant number of private firms, cannot issue preferred stock. They can issue multiple classes of common stock, however.

The weighted average cost of capital (WACC) is calculated as the weighted average of the costs of the components of a firm's capital structure. The WACC for a firm that has debt $(d)$, equity $(e)$ and preferred equity $(p e)$ is defined by Equation 5.1.

$$
\begin{equation*}
k_{\mathrm{wacc}}=w_{d} \times k_{d} \times(1-T)+w_{e} \times k_{e}+w_{p e} k_{p e} \tag{5.1}
\end{equation*}
$$

where $w=$ the market value of each component of the firm's capital structure divided by the total market value of the firm
$k=$ the cost of capital for each component of the capital
structure
$T=$ the tax rate

The WACC is used in conjunction with the discounted free cash flow method, which was used in Chapter 4 to value Tentex. The sections that follow first focus on estimating the cost of equity capital. Although there are two competing theories of estimating the cost of capital, and equity capital in particular, the capital asset pricing model (CAPM) and arbitrage pricing theory (APT), this chapter focuses on an adjusted version of the CAPM known as the buildup method. The major reason is that this model is the one most often used by valuation analysts when estimating the cost of equity capital for private firms. Finally, we demonstrate how to estimate the cost of debt and preferred stock for private firms.

## THE COST OF EQUITY CAPITAL

The basic model for estimating a firm's cost of capital is a modified version of the CAPM, as shown in Equation 5.2.

$$
\begin{equation*}
k_{s}=k_{r f}+\operatorname{beta}_{s}\left[\mathrm{RP}_{m}\right]+\operatorname{beta}_{s-1}\left[\mathrm{RP}_{m}\right]_{-1}+\mathrm{SP}_{s}+\operatorname{FSRP}_{s} \tag{5.2}
\end{equation*}
$$

$$
\text { where } \begin{aligned}
k_{s} & =\text { cost of equity for firm } s \\
k_{r f} & =\text { the } 10 \text {-year risk-free rate } \\
\text { beta }_{s} & =\text { systematic risk factor for firm } s \\
\text { beta }_{s-1} & =\text { betas in the previous period } \\
\mathrm{RP}_{m} & =\text { additional return investors require to invest in a } \\
& \text { diversified portfolio of financial securities rather than the } \\
& \text { risk-free asset } \\
\mathrm{RP}_{(m-1)} & =\mathrm{RP} \text { in the previous period } \\
\mathrm{SP}_{s} & =\text { additional return investors require to invest in firm } \\
& s \text { rather than a large capitalization firm } \\
\mathrm{FSRP}_{s}= & \text { additional return an owner of firm } s \text { requires due to the } \\
& \text { fact that the owner does not have the funds available to } \\
& \text { diversify away the firm's unique, or specific, risk }
\end{aligned}
$$

To estimate the cost of equity capital for firm $s$, values for the parameters in Equation 5.1 need to be developed. Ibbotson Associates is the source of several of these parameters. ${ }^{1}$ The equity risk premium, $\mathrm{RP}_{m}$, is calculated as the difference between the total return on a diversified portfolio of stock of large companies as represented by the NYSE stock return index, for example, and the income return from a Treasury bond that has

TABLE 5.1 Equity Risk Premiums for Various Time Periods

| Time Period: Start Date | Period Dates | Equity Risk <br> Premium |
| :--- | :---: | :---: |
| Depression | $1932-2001$ | $8.10 \%$ |
| War | $1942-2001$ | $8.30 \%$ |
| Recession | $1982-2001$ | $8.00 \%$ |
| Average |  | $8.13 \%$ |
| Business cycle peak | $1962-2001$ | $4.80 \%$ |
| Business cycle peak | $1972-2001$ | $5.50 \%$ |
| Average |  | $5.15 \%$ |
|  |  | $6.64 \%$ |
| Overall average | $1926-2001$ | $7.40 \%$ |
| Long-term risk premium |  |  |

20 years to mature. The income return is defined as the portion of the total return that comes from the bond's coupon payment. Table 5.1 shows the realized average equity risk premium through 2001 for different starting dates.

Table 5.1 indicates that the equity risk premium varies over different time spans. The risk premium required in Equation 5.1 equates to what an analyst would expect the risk premium to average over an extended future period. It appears from the preceding data that the risk premium values are higher when the starting point is in a recession or slow-growth year (e.g., 1932, 1982), and smaller when the starting point is in a high-growth year, relatively speaking (e.g., 1962, 1972). Ideally, the risk premium used in Equation 5.1 should reflect a normal starting and ending year rather than an extended period dominated by a unique set of events, like a war, for example.

## CALCULATING BETA FOR A PRIVATE FIRM

Beta is a measure of systematic risk. Using regression techniques, one can estimate beta for any public firm by regressing its stock returns on the returns earned on a diversified portfolio of financial securities. For a private firm, this is not possible; the beta must be obtained from another source. The steps taken to calculate a private firm beta can be summarized as follows:

- Estimate the beta for the industry that the firm is in.
- Adjust the industry beta for time lag.
- Adjust the industry beta for the size of the target firm.
- Adjust the industry beta for the capital structure of the target firm.


## Estimating the Industry Beta

Research indicates that firm betas are more variable than industry betas, and therefore systematic risk of a firm may be better captured using an industry proxy. Ibbotson Associates, a primary data source for industry betas, notes:

> Because betas for individual companies can be unreliable, many analysts seek to calculate industry or peer group average betas to determine the systematic risk inherent in a given industry. In addition, industry or peer group averages are commonly used when the beta of a company or division cannot be determined. A beta is either difficult to determine or unattainable for companies that lack sufficient price history (i.e., non-publicly traded companies, divisions of companies, and companies with short price histories). Typically, this type of analysis involves the determination of companies competing in a given industry and the calculation of some sort of industry average beta. ${ }^{2}$

Ibbotson Associates has developed betas by industry, as defined by SIC code. Firms included in a specific industry must have at least 75 percent of their revenues in the SIC code in which they are classified. Table 5.2 shows the Ibbotson data for SIC 3663, radio and television broadcasting equipment. ${ }^{3}$

The betas shown are for two size classes, an industry composite, which is akin to a weighted average of the firm betas that make up the industry, and the median industry beta. Ibbotson Associates also calculates levered and unlevered versions of the betas in Table 5.2. Since most firms in Ibbotson's data set are in multiple industries, Ibbotson has developed a process that captures this effect. Ibbotson refers to the product of this analysis as the adjusted beta. ${ }^{4}$ The levered industry beta reflects the actual capital structure of the firms included in the industry, some of which have debt in their capital structure. By removing the influence of financial risk due to debt in the capital structure, one obtains the unlevered industry beta. This beta reflects only systematic business risk and not the financial risk that emerges because firms in the industry have debt in their capital structures. We return to the relationship between levered and unlevered betas in a subsequent section. For the moment we focus on the nonleverage adjustments that need to be made to the unlevered industry beta before it can used to estimate the cost of equity capital for a private firm.

TABLE 5.2 Statistics for SIC Code 3663

| This Industry Comprises 40 Companies |  |  |  |
| :---: | :---: | :---: | :---: |
| Sales (\$ Millions) |  | Total Capital (\$ Millions) |  |
| Total | 34,907.0 | Total | 34,170.0 |
| Average | 872.7 | Average | 854.3 |
| Three Largest Companies |  | Three Largest Companies |  |
| Motorola Inc. | 30,004.0 | Motorola Inc. | 28,853.9 |
| Scientific-Atlanta Inc. | 1,671.1 | Scientific-Atlanta Inc. | 2,110.7 |
| Allen Telecom Inc. | 417.0 | Tekelec | 648.6 |
| Three Smallest Companies |  | Three Smallest Companies |  |
| Amplidyne Inc. | 2.2 | Electronic System Tech Inc. | 1.9 |
| Simtrol Inc. | 1.9 | Technical Communications CP | 1.1 |
| Electronic System Tech Inc. | 1.3 | Amplidyne Inc. | 0.8 |


|  | Levered Betas |  |  |
| :--- | :---: | :---: | :---: |
|  | Raw Beta | Adjusted Beta |  |
| Adjusted Beta |  |  |  |
| Median | 1.47 | 1.76 | 0.81 |
| SIC composite | 1.56 | 1.66 | 1.29 |
| Large composite | 1.53 | 1.63 | 1.26 |
| Small composite | 1.87 | 2.01 | 1.87 |

While Ibbotson has estimated betas for many industries, the industry coverage is by no means complete. Most private firms operate in detailed segments of industries covered by Ibbotson at a more aggregate level. The valuation analyst has three choices when the firm being valued is in an industry segment not covered by publicly available databases like Ibbotson Associates. First, one can choose to use a beta for a more aggregate industry that is related to the industry in which the target firm operates. The second choice is to assume the relevant beta is unity, since research suggests that betas drift toward the riskiness of the overall market. The third choice is to develop a model that estimates the beta for the disaggregate sector.

To see how one might implement this last option, we consider a version of the basic CAPM regression equation used to estimate beta, Equation 5.3.

$$
\begin{equation*}
k_{I}=\alpha_{I}+\operatorname{beta}_{I} k_{m}+\varepsilon_{I} \tag{5.3}
\end{equation*}
$$

```
where \(\quad k_{I}=\) the return on a portfolio of firms operating in industry \(I\)
    \(k_{m}=\) the return on a broad market index (e.g., New York Stock
        Exchange Index)
    beta \(_{I}=\) the measure of systematic risk for industry \(I\)
    \(\alpha_{I}=\) a constant term
    \(\varepsilon_{I}=\) the regression error term
```

An analogous relationship to Equation 5.3 can be written where the percent change in operating earnings before tax for a segment of industry $I$, denoted as $\% \mathrm{PTI}_{i}$, is regressed against the percentage change in operating earnings for the overall economy, $\% \mathrm{PTI}_{e}$, as shown in Equation 5.4.

$$
\begin{equation*}
\% \mathrm{PTI}_{i}=\partial_{i}+\text { beta }_{i} \% \mathrm{PTI}_{e}+\mu_{i} \tag{5.4}
\end{equation*}
$$

We now assume that the beta for segment $i$ is related to the beta of its more aggregate industry sector $I$ plus a constant term related to the difference in systematic risk between the aggregate industry and its segment, as shown in Equation 5.5.

$$
\begin{equation*}
\operatorname{beta}_{i}=\operatorname{beta}_{I}+c_{i} \tag{5.5}
\end{equation*}
$$

Substituting Equation 5.4 into Equation 5.5 and noting that beta ${ }_{I}$ can be obtained from a source like Ibbotson gives rise to Equation 5.6.

$$
\begin{equation*}
\% \mathrm{PTI}_{i}-\text { beta }_{I} \times \% \mathrm{PTI}_{e}=\partial_{i}+c_{i} \times \% \mathrm{PTI}_{e}+\mu_{i} \tag{5.6}
\end{equation*}
$$

Axiom Valuation Solutions has constructed a time series for \%PTI for 700 industries defined by SIC. ${ }^{5}$ This data set was developed from multiple government sources. Using Axiom's data, Equation 5.6 was estimated. The final value of $c_{i}$ was obtained using a two-stage procedure. This is done because many of the initial values of $c_{i}$ from estimating Equation 5.6 were often implausibly high or low, and in some cases statistically insignificant. Such divergence is not surprising because the underlying Ibbotson and Axiom data come from different sources. To reduce the divergence and still capture the differential variability of beta within detailed industry segments, a second-stage regression was estimated for which the estimated industry $c_{i}$ was the dependent variable, and $c_{i}$ was then regressed against the aggregate industry beta and the standard deviation of the growth in industry-segment operating earnings. Equation 5.7 was the equation estimated, and Table 5.3 shows the results of this second-stage regression.

$$
\begin{equation*}
c_{i}=d_{0}+d_{1} \times \text { beta }_{I}+d_{2} \times \operatorname{std} \% \operatorname{PTI}_{i}+\theta_{i} \tag{5.7}
\end{equation*}
$$

TABLE 5.3 Beta Decomposition Equation

| Summary Output |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Regression Statistics |  |  |  |  |  |
| Multiple R |  | 0.546048696 |  |  |  |
| R square |  | 0.298169178 |  |  |  |
| Adjusted R square |  | 0.296155317 |  |  |  |
| Standard error |  | 1.827726737 |  |  |  |
| Observations |  | 700 |  |  |  |
| ANOVA |  |  |  |  |  |
| df |  | SS | MS | F | Significance F |
| Regression <br> Residual <br> Total | 2 | $\begin{aligned} & 989.2034441 \\ & 2328.387762 \\ & 3317.591206 \end{aligned}$ | $\begin{aligned} & 494.6017221 \\ & 3.340585025 \end{aligned}$ | 148.0584144 | $2.58229 \mathrm{E}-54$ |
|  | 697 |  |  |  |  |
|  | 699 |  |  |  |  |
|  | Coefficients | Standard Error | t-Stat | P-value | Lower 95\% |
| Intercept | -0.300591958 | 8.156793904 | -1.917115082 | 0.055631815 | -0.60843667 |
| Beta | -0.520569128 | 0.201171257 | -2.587691385 | 0.009863351 | -0.915543078 |
| Standard deviation | 3.584498155 | - 0.210456798 | 17.03199038 | 1.197E-54 | 3.171293237 |

The regression results indicate that the coefficients are statistically significant. The explanatory power of the equation indicates that 30 percent of the variance in $c_{i}$ is explained by the estimated cross-section relationship. Using the results of this two-step procedure, we can estimate beta ${ }_{i}$ as Equation 5.8

$$
\begin{equation*}
\text { beta }_{i}=-0.30+(1-0.52) \times \operatorname{beta}_{I}+3.58 \times \operatorname{std} \% \mathrm{PTI}_{i} \tag{5.8}
\end{equation*}
$$

Now let us consider an example of how to use Equation 5.8. Assume we need to calculate beta for a firm in SIC 3317 (steel pipes and tubes), but have only the median unlevered beta for SIC 331 (steelworks, blast furnaces, and rolling and finishing mills), which is equal to 0.44 . An approximation to the unlevered median industry beta for SIC 3317 is 0.52 as shown here.

$$
\text { beta }_{3317}=-0.30+(1-0.52) \times 0.44+3.58 \times(.017)=0.52
$$

## Adjusting Beta for Size

The next step in estimating beta relates to adjusting the estimated median beta for size. Ibbotson and others have noticed that beta of small-company
portfolios, though higher than for large-company portfolios were, nevertheless, not high enough to explain all of the excess return historically found in small stocks. Since private firms are generally smaller than the smallest public firms, this problem is likely to be magnified for them. One explanation for the small-firm beta bias is that small-firm stocks are often infrequently traded, so their share prices do not always move with the overall market. This would result in an estimated beta that would be biased downward. One way to remove or limit this bias is to estimate a lagged version of the capital asset pricing model.

$$
\begin{equation*}
k_{s}-k_{r f}=\partial_{s}+\operatorname{beta}_{s}\left[\mathrm{RP}_{m}\right]+\operatorname{beta}_{s-1}\left[\mathrm{RP}_{m}\right]_{-1}+\varepsilon_{s} \tag{5.9}
\end{equation*}
$$

Sumbeta is the term for beta $_{s}+$ beta $_{s-1}$. Ibbotson Associates has estimated the sumbeta for 10 different-size classes based on market capitalization. Axiom Valuation Solutions has converted capitalization class sizes to sales class sizes and extended the class range to 15 beta and sumbeta-size classes. Table 5.4 shows the results of this analysis.

Now let us use the data in Table 5.4 to adjust the estimated beta for steel pipes and tubes. First note the relationship in Equation 5.10. The first term of the equation is the size factor. Note that it is symmetrical about the median value of 1.31 shown in the last row of Table 5.4. The second term is a factor that when multiplied by the size beta will yield the sumbeta. If we assume that Equation 5.10 holds approximately at the industry level, then we can use the values in the last column of Table 5.4 to adjust the median industry beta for target firm size and the beta lag effect.

$$
\begin{equation*}
\frac{\text { Size beta }}{\text { Median beta }} \times \frac{\text { sumbeta }}{\text { size beta }}=\frac{\text { sumbeta }}{\text { median beta }} \tag{5.10}
\end{equation*}
$$

An example will be helpful here. Assume one desires to estimate beta for a steel pipe and tube firm that has sales of slightly less than $\$ 1$ million. The median beta for this industry was estimated earlier to be 0.52 . When this value is multiplied by 1.399 , which is the factor for firms with less than $\$ 1$ million in revenue, the beta is increased to 0.73 , which represents an increase in systematic risk of 40 percent.

## Impact of Leverage on a Firm's Beta

Once the unlevered beta has been calculated, it can then be adjusted for the leverage of the firm being valued. To understand the impact of leverage on a firm's beta, we note the basic accounting identity shown in Equation 5.11.

$$
\begin{equation*}
\text { Assets }=\text { equity }+ \text { debt } \tag{5.11}
\end{equation*}
$$

TABIE 5.4 Beta Size Adjustment

| Size Beta |  | Sum Size Beta |  | Sales |  | Ratio of Sumbeta to Size Beta |  | Size Factor: Size <br> Beta/Median Size Beta | Beta Sum, Size $\times$ Size Factor |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Percen |  | Percentile |  |  | Percentile | Percentile |  | Percentile | Percentile |  |
| 1-largest | 0.9100 | 1-largest | 0.9100 | 1-largest | \$22,225,812,786.89 | 1-largest | 1 | 1-largest 0.69465649 | 1 | 0.6946565 |
| 2 | 1.0400 | 2 | 1.0600 | 2 | \$3,322,210,029.59 | 2 | 1.019231 | 0.79389313 | 2 | 0.8091603 |
| 3 | 1.0900 | 3 | 1.1300 | 3 | \$1,954,637,143.27 | 3 | 1.036697 | 30.83206107 | 3 | 0.8625954 |
| 4 | 1.1300 | 4 | 1.1900 | 4 | \$1,138,054,576.81 | 4 | 1.053097 | 0.86259542 | 4 | 0.9083969 |
| 5 | 1.1600 | 5 | 1.2400 | 5 | \$711,964,358.60 | 5 | 1.068966 | 0.88549618 | 5 | 0.9465649 |
| 6 | 1.1800 | 6 | 1.3000 | 6 | \$508,957,368.04 | 6 | 1.101695 | 60.90076336 | 6 | 0.9923664 |
| 7 | 1.2400 | 7 | 1.3800 | 7 | \$321,128,186.91 | 7 | 1.112903 | 70.94656489 | 7 | 1.0534351 |
| 8 | 1.2800 | 8 | 1.4800 | 8 | \$199,600,897.93 | 8 | 1.15625 | 0.97709924 | 8 | 1.129771 |
| 9 | 1.3400 | 9 | 1.5500 | 9 | \$185,000,000.00 | 9 | 1.156716 | 1.02290076 | 9 | 1.1832061 |
| 10a | 1.4300 | 10a | 1.7100 | 10a | \$120,121,611.60 | 10a | 1.195804 | 10a 1.09160305 | 10 | 1.3053435 |
| 10b | 1.4100 | 10b | 1.7100 | 10b | \$41,913,488.23 | 10b | 1.212766 | 10 b 1.07633588 | 11 | 1.3053435 |
| 11 | 1.4239 | 11 | 1.7347 | 11 | \$31,900,000.00 | 11 | 1.218278 | 111.08693956 | 12 | 1.3241945 |
| 12 | 1.4378 | 12 | 1.7594 | 12 | \$21,900,000.00 | 12 | 1.223683 | 121.09754323 | 13 | 1.3430455 |
| 13 | 1.4517 | 13 | 1.7841 | 13 | \$11,900,000.00 | 13 | 1.228985 | 131.10814691 | 14 | 1.3618965 |
| 14 | 1.4656 | 14 | 1.8088 | 14 | \$1,000,000.00 | 14 | 1.234187 | 141.11875059 | 15 | 1.3807474 |
| 15 | 1.4795 | 15 | 1.8335 | 15 | >\$1,000.000 | 15 | 1.239291 | 151.12935427 | 16 | 1.3995984 |
| Median | 1.3100 |  |  |  |  |  |  |  |  |  |

This accounting identity implies that the firm's asset beta is equal to the weighted average of the betas of the components of its capital structure, which in this case is made up of debt $D$ and equity $E$. The equity and debt weights are the percent of the firm's assets financed with debt and equity, respectively, Equations 5.12 and 5.13.

$$
\begin{gather*}
\operatorname{beta}_{a}=\left(\frac{E}{D+E}\right) \text { beta }_{e}+\left(\frac{D}{D+E}\right) \text { beta }_{d}  \tag{5.12}\\
\text { beta }_{e}=\operatorname{beta}_{a}+\frac{D}{E}\left(\operatorname{beta}_{a}-\operatorname{beta}_{d}\right) \tag{5.13}
\end{gather*}
$$

Beta ${ }_{a}$ is an indicator of the risk of the operating assets of the business. This beta is unrelated to how the assets of the firm are financed, and hence it is equivalent to the firm's unlevered beta, beta ${ }_{u}$, shown in Equation 5.14. Noting that interest is a tax-deductible expense to the firm, and $T$ being the tax rate, the relationship between the levered and unlevered beta can be written as shown in Equation 5.14.

$$
\begin{equation*}
\text { beta }_{l}=\operatorname{beta}_{u} \times\left[1+\left(\frac{D}{E}\right) \times(1-T)\right]-\operatorname{beta}_{d} \times\left[\frac{D \times(1-T)}{E}\right] \tag{5.14}
\end{equation*}
$$

If the debt beta is taken to be zero, Equation 5.14 can be written as Equation 5.15 , which is known as the Hamada equation. ${ }^{6}$

$$
\begin{equation*}
\text { beta }_{l}=\operatorname{beta}_{u} \times\left[1+\left(\frac{D}{E}\right) \times(1-T)\right] \tag{5.15}
\end{equation*}
$$

Now let us calculate the levered beta assuming the size-adjusted unlevered beta is 0.73 . If the market value of debt is $\$ 300,000$, and the market value of equity is $\$ 700,000$, then we can use Equation 5.16 to calculate the levered beta.

$$
\begin{equation*}
\text { beta }_{l}=0.73 \times\left[1+\left(\frac{\$ 300}{\$ 700}\right) \times(1-0.4)\right]=0.73 \times(1+0.26)=0.92 \tag{5.16}
\end{equation*}
$$

A beta value of 0.92 represents the levered beta adjusted for size that should be used in Equation 5.1 to calculate the equity cost of capital. Note that this beta is in excess of 100 percent larger than the initial unlevered beta of 0.44 . This difference effectively means that the cost of equity capital will be significantly higher than would be the case if the beta were not adjusted for industry segment, size, and the beta lag effect.

## Size Premium

Ibbotson has shown that even after accounting for the unlevered beta size adjustment, small firms still earn excess returns, although these returns are

TABLE 5.5 Size Premiums for Size Premium Beta and Size Premium Sumbeta

| Size Class | Sales | Size Premium <br> (Beta) | Size Premium <br> (Sumbeta) |
| :--- | ---: | ---: | :---: |
| 1-largest | $\$ 22,225,812,786.89$ | $0.16 \%$ | $-0.34 \%$ |
| 2 | $\$ 3,322,210,029.59$ | $0.95 \%$ | $0.34 \%$ |
| 3 | $\$ 1,954,637,143.27$ | $1.15 \%$ | $0.43 \%$ |
| 4 | $\$ 1,138,054,576.81$ | $1.56 \%$ | $0.60 \%$ |
| 5 | $\$ 711,964,358.60$ | $1.83 \%$ | $0.79 \%$ |
| 6 | $\$ 508,957,368.04$ | $2.03 \%$ | $0.72 \%$ |
| 7 | $\$ 321,128,186.91$ | $1.99 \%$ | $0.52 \%$ |
| 8 | $\$ 199,600,897.93$ | $2.66 \%$ | $0.79 \%$ |
| 9 | $\$ 185,000,000.00$ | $3.32 \%$ | $1.38 \%$ |
| $10-$ smallest | $\$ 120,121,611.60$ | $6.76 \%$ | $4.21 \%$ |
| Mid-cap, 3-5 |  | $1.37 \%$ | $0.53 \%$ |
| Low-cap, 6-8 |  | $2.13 \%$ | $0.65 \%$ |
| Micro-cap, 9-10 |  | $4.42 \%$ | $2.28 \%$ |

smaller when the sumbeta adjusted for size rather than simple size adjusted betas are used. Table 5.5 shows the differences in the size premiums when beta and sumbeta are used in the calculations. ${ }^{7}$

The size premium based on beta indicates that size is an important factor for firms with sales of less than $\$ 22$ billion dollars. When the sumbeta is used, the size premium shows little variation through size class 8 . The risk premium then rises significantly between class 8 and class 10 . For example, when sales are about $\$ 200$ million, the size premium is 0.79 percent, which is not much greater than for larger size classes. However, when sales decline by $\$ 80$ million, the size premium increases to 4.21 percent. This suggests that the risk premium is likely to rise more than proportionately in relation to the decline in sales the lower the sales level, indicating that the risk premium for firms below $\$ 50$ million in sales, for example, is likely to be quite large. The implication of this is that a valuation analyst using the smallest Ibbotson size premium when estimating the cost of capital for a firm that has $\$ 10$ million in sales is more than likely to estimate a cost of capital that is too low, thereby producing an income-based valuation that is correspondingly too large.

How might a valuation analyst adjust the size premium for a small firm? In the absence of any additional information, one could increase the size premium by 3.42 percent $(4.21 \%-0.79 \%)$ for each $\$ 80$ million decrement in sales. This would imply that a firm with $\$ 10$ million in sales would have a size premium equal to 8.91 percent $(4.21 \%+3.42 \%+(\$ 30 \mathrm{M} / \$ 80 \mathrm{M}) \times 3.42 \%)$. Because the relationship between the size-risk premium and sales size is likely to be nonlinear when sales are lower than $\$ 100$ million dollars, the suggested
correction may still understate the cost of capital for smaller private firms. At the moment, however, this likely the best that can be done to correct the cost-of-equity calculation for small firms.

## The Firm-Specific Risk Premium

In standard finance theory, the equity cost of capital does not reflect firmspecific risk, because it is assumed that the risk unique to a firm can be diversified away. Thus, if the investor does not have to bear the risk, then the financial markets will not reward the investor for taking it. In estimating the cost of capital for a private firm, it is generally assumed that the owners cannot diversify away from the unique risk that the firm represents, and thus anybody desiring to purchase the firm would incorporate a premium to reflect this fact.

Firm-specific risk as it is generally understood refers to business risk that is associated with the unique characteristics of the firm. Table 5.6 shows some of the factors that would ordinarily be considered when assessing the magnitude of firm-specific risk. In this example, high risk, moderate risk, and low risk are given five points, three points, and one point, respectively. The weights given to each of the factors are arbitrary, although their relative values generally conform to the relative importance of the factors that most impact private firms. Many private firms have a great reliance on key personnel such that, if they were not available, the success of the business would be compromised. Hence, one would think that the weight given to this factor should be greater than 20 percent. It is not because this risk can be partially protected against through the purchase of key-person insurance. Hence, in part or in whole, the risk is diversifiable, thus the weighting reflects this possibility.

Now that the risk factors have been assessed and points determined, how does one go about relating the point total to the incremental return that a purchaser of the firm would require. As a matter of practice, the valuation analyst may have a rule that says if the point total is greater than 4 then the firm-specific risk premium is 5 percent. If the point total is between 3.1 and 3.9 , then the risk premium would be set at 4 percent and so on. However, such a scheme is arbitrary.

To get an idea about the size of the firm-specific risk premium, one can review the returns earned on venture-capital funds. Venture capitalists raise money from diversified investors, pay a return consistent with the investment's systematic risk, and capture the resulting excess return. This additional return is what venture capitalists require to accept firm-specific risk of the firms in their funds.

Gompers and Lerner measure returns for a single private equity group from 1972 to 1997. Using a version of the CAPM, they find that additional

TABLE 5.6 Factors That Determine Firm-Specific Risk
Firm-Specific Risk Matrix

| Risk Concept | Measurement | Assessment | Factor Weight | Weighted <br> Assessment |
| :---: | :---: | :---: | :---: | :---: |
| Business stability | How long has the company been profitable? <br> 1-3 years—High risk: 5 <br> 4-6 years—Moderate risk: 3 <br> More than 6 years-Low risk: 1 | High risk: 5 | 10.00\% | 0.50 |
| Business transparency | Does the firm produce an audited financial statement at least once a year? <br> Yes—Low risk: 5 <br> No—High Risk: 1 | Low risk: 1 | 10.00\% | 0.50 |
| Customer concentration | Does the firm receive more than $30 \%$ of its revenue from less than 5 customers? <br> Yes—High risk: 5 <br> No-Low risk: 1 | High risk: 5 | 25.00\% | 1.25 |
| Supplier reliance | Can the firm change suppliers without sacrificing product/service quality or increasing costs? <br> Yes—Low risk: 1 <br> No—High risk: 5 | High risk: 5 | 10.00\% | 0.50 |
| Reliance on key people | Are there any personnel critical to the success of the business that cannot be replaced in a timely way at the current market wage? <br> Yes-High risk: 5 <br> No—Low risk: 1 | High risk: 5 | 20.00\% | 1.00 |
| Intensity of competition | What is the intensity of firm competition? <br> Very intense—High risk: 5 <br> Modestly intense-Moderate risk: 3 <br> Not very intense-Low risk: 1 | High risk: 5 | 25.00\% | 1.25 |
| Sum |  |  | 100.00\% | 5.00 |

return earned above the CAPM return was about 8 percent. ${ }^{8}$ Cochrane studied all venture investments in the VentureOne database from 1987 through June 2000. ${ }^{9}$ After adjusting the data for selection bias, he estimates an arithmetic average annualized return of 57 percent, with an arithmetic standard deviation of 119 percent. The beta of these funds was about unity, implying a return in excess of CAPM in the neighborhood of 40 percent. This return is likely to be too high, since it is not net of fees and other compensation that venture capitalists ordinarily receive. The return standard deviation also suggests a great deal of variability. Despite these shortcomings, it appears that firm-specific risk is significant and should be part of any cost of equity capital calculation.

## THE COST OF DEBT

Like public firms, private firms have debt on the balance sheet. For newly issued debt at par, the cost is simply the coupon rate, or if it is bank debt, it is typically some function of the prime rate. Estimating the cost of debt becomes somewhat more difficult when the analyst needs to calculate the current cost of previously issued debt. This exercise can be carried out by undertaking a credit analysis of the firm in much the same way a bank credit analyst might do. One model that is very useful for this purpose is Altman's Z score model. ${ }^{10}$ The steps in determining the cost of a private firm's debt using this model are:

- Estimate the firm's Z score using the Altman model.
- Convert the Z score to a debt rating.
- Determine the cost of debt for a given maturity as the rate on a Treasury security of equivalent maturity plus the expected yield spread of equivalent debt relative to the rate on the Treasury security.
- Add an additional risk premium to reflect firm size.

The Z score model for private firms is given by Equation 5.17.

$$
\begin{equation*}
Z=0.717 \times X_{1}+0.847 \times X_{2}+3.107 \times X_{3}+0.42 \times X_{4}+0.998 \times X_{5} \tag{5.17}
\end{equation*}
$$

where $X_{1}=\frac{\text { (current assets }- \text { current liabilities) }}{\text { total assets }}$

$$
\begin{aligned}
& X_{2}=\frac{\text { retained earnings }}{\text { total assets }} \\
& X_{3}=\frac{\text { earnings before interest and taxes }}{\text { total assets }}
\end{aligned}
$$

$$
\begin{aligned}
& X_{4}=\frac{\text { book value of equity }}{\text { total liabilities }} \\
& X_{5}=\frac{\text { sales }}{\text { total assets }}
\end{aligned}
$$

Table 5.7 shows the relationship between the firm's debt rating and its Z score by maturity of debt.

Using the Z score model, we can now calculate the cost of debt for Tentex, the private firm introduced in Chapter 4. Table 5.8 reproduces Tentex's balance sheet. Table 5.9 shows the calculation of Tentex's Z score. Tentex's Z score is 3.1 , which translates to debt rated between C and $\mathrm{B} 3 / \mathrm{B}$ - (refer to Table 5.7). The weighted average maturity of Tentex's debt is about 10 years. If the 10 -year Treasury note rate is 4.68 percent, then based on Table 5.9 , the rate on Tentex debt should be this rate plus 775 basis points (see Table 5.7), or 12.43 percent.

The 12.3 percent represents the rate that Tentex would be charged based solely on an analysis of its credit risk. The effective rate is likely to be larger, however, since loans to private businesses are typically secured by

TABLE 5.7 Relationship between, Z Score, Debt Rating, and Yield Spread

| Debt Rating | Z-Score | Yield Spreads over like Maturity Treasuries: Basis Points |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Maturity in Years |  |  |  |  |  |  |
|  |  | 1 | 2 | 3 | 5 | 7 | 10 | 30 |
| Aaa/AAA | 8.15 | 5 | 10 | 15 | 22 | 27 | 30 | 55 |
| Aa1/AA+ | 7.6 | 10 | 15 | 20 | 32 | 37 | 40 | 60 |
| Aa2/AA | 7.3 | 15 | 25 | 30 | 37 | 44 | 50 | 65 |
| Aa3/AA- | 7 | 20 | 30 | 35 | 45 | 54 | 60 | 70 |
| A1/A+ | 6.85 | 30 | 40 | 45 | 60 | 65 | 70 | 85 |
| A2/A | 6.65 | 40 | 50 | 57 | 67 | 75 | 82 | 89 |
| A3/A- | 6.4 | 50 | 65 | 70 | 80 | 90 | 96 | 116 |
| Baa1/BBB+ | 6.25 | 60 | 75 | 90 | 100 | 105 | 114 | 135 |
| Baa2/BBB | 5.85 | 75 | 90 | 105 | 115 | 120 | 129 | 155 |
| Baa3/BBB- | 5.65 | 85 | 100 | 115 | 125 | 133 | 139 | 175 |
| Ba1/BB+ | 5.25 | 300 | 300 | 275 | 250 | 275 | 225 | 250 |
| Ba2/BB | 4.95 | 325 | 400 | 425 | 375 | 325 | 300 | 300 |
| Ba3/BB- | 4.75 | 350 | 450 | 475 | 400 | 350 | 325 | 400 |
| B1/B+ | 4.5 | 500 | 525 | 600 | 425 | 425 | 375 | 450 |
| B2/B | 4.15 | 525 | 550 | 600 | 500 | 450 | 450 | 725 |
| B3/B- | 3.75 | 725 | 800 | 775 | 750 | 725 | 775 | 850 |
| Caa/CCC | 2.5 | 1500 | 1600 | 1550 | 1400 | 1300 | 1375 | 1500 |

Source: Altman and BondsOnline Corporate Yield-Spread Matrix.

TABLE 5.8 Tentex's Balance Sheet

| Row | Concepts | 2003 | 2002 |  | Change:$2003 / 2002$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Assets |  |  |  |  |
| 1 | Cash | \$220,000 |  | \$187,000 |  |
| 2 | Cash required for operations | \$71,251 |  | \$64,126 |  |
| 3 | Excess cash | \$148,749 |  | \$122,874 |  |
| 4 | Accounts receivable | \$356,256 |  | \$302,817 |  |
| 5 | Inventories | \$890,639 |  | \$846,107 |  |
| 6 | Other current assets | \$0 |  | \$0 |  |
| 7 | Total current assets | \$1,686,895 |  | \$1,522,924 |  |
| 8 | Gross plant and equipment | \$5,343,834 |  | \$5,076,642 |  |
| 9 | Accumulated depreciation | \$3,730,729 |  | \$3,480,729 |  |
| 10 | Net fixed capital | \$1,613,105 |  | \$1,595,914 |  |
| 11 | Total assets | \$3,300,000 |  | \$3,118,838 |  |
| 12 | Liabilities and Equity |  |  |  |  |
| 13 | Short-term debt and current portion of long-term debt | \$200,000 |  | \$190,000 |  |
| 14 | Accounts payable | \$178,128 |  | \$160,315 |  |
| 15 | Accrued liabilities | \$50,000 |  | \$42,500 |  |
| 16 | Total current liabilities | \$428,128 |  | \$392,815 |  |
| 17 | Long-term debt | \$490,000 |  | \$454,151 |  |
| 18 | Other long-term liabilities | \$0 |  | \$90,000 |  |
| 19 | Deferred income taxes | \$0 |  |  |  |
| 20 | Total shareholder equity | \$2,381,872 |  | \$2,181,872 |  |
| 21 | Total liabilities and equity | \$3,300,000 |  | \$3,118,838 |  |
| 22 | Working capital | \$890,018 | \$0 | \$820,235 | \$69,783 |
| 23 | Net fixed capital | \$1,613,105 | \$0 | \$1,595,914 | \$17,192 |
| 24 | Net capital requirements |  |  |  | \$86,974 |
| 25 | NOPAT | \$362,201 |  |  |  |
| 26 | Interest expense | \$55,800 |  |  |  |
| 27 | Income available to shareholders and creditors | \$418,001 |  |  |  |
| 28 | Free cash flow to the firm (row 27-row 24) | \$331,026 |  |  |  |

TABLE 5.9 Tentex Z Score

|  | (Current Assets <br> Current | Accumulated <br> Liabilities)/ <br> Assets | Retained <br> Earnings/ <br> Assets* | EBIT// <br> Assets | Book Value <br> Equity/Total <br> Liabilities |
| :--- | :---: | :---: | :---: | :---: | :---: | | Sales/ |
| :---: |
| Assets |

*Accumulated retained earnings is 20 percent of shareholder equity.
business assets and/or the personal guarantee of the owners. In addition, some lenders require an additional yield depending on firm size. The logic behind this premium is that smaller firms are inherently more risky than equivalent larger firms, even when their credit risk profiles are equal. This phenomena is consistent with the way the equity markets assess systematic risk, with smaller firms having a greater cost of equity capital than their larger-firm counterparts, all else equal (other than firm size).

Although we are not aware of evidence of this size bias, the SBA 7(a) program offers some insight on what the size premium might be. The 7(a) program requires partner banks to set small business loan rates based on the prime rate plus anywhere between 2.75 and 4.75 percent. While the SBA does not refer to these differentials as size premiums, the fact that the SBA guarantees a portion of the loan, up to 85 percent, and requires that borrowers personally guarantee the loan, in addition to the firm providing collateral, suggests that these differentials in part or in total are related to firm size. ${ }^{11}$ In Tentex's case, if it refinanced its $\$ 690,000$ in loans outstanding based on the preceding facts, the likelihood is that the market rate would be in the neighborhood of 15.18 percent $(12.43 \%+2.75 \%)$ to 17.18 percent $(12.43 \%+4.75 \%)$.

Based on an interest rate of 15.18 percent ( $7.6 \%$ compounded semiannually) and interest payments over a 10 -year period of $\$ 55,000$ per year, principal repayment of $\$ 690,000$, the market value of Tentex's debt can be calculated using Equation 5.18.

$$
\begin{equation*}
D_{\mathrm{TENTEX}}=\sum_{t=1}^{20} \frac{(\$ 27,500)_{t}}{(1+0.076)^{t}}+\frac{\$ 717,500}{(1+0.076)^{10}}=\$ 438,179 \tag{5.18}
\end{equation*}
$$

If the interest rate were 17.18 percent, the market value of Tentex's debt would be $\$ 391,303$. When using the discounted free cash flow model, the market value of debt would be calculated in this way. ${ }^{12}$
TABLE 5.10 Preferred Stock Returns versus Common Stock Returns

| Preferred Stock | Average Monthly <br> Return: $1998 — 01.2003$ | Common Stock | Average Monthly <br> Return: |  |
| :--- | :--- | :--- | :--- | :---: |
| 1 | FORD MOTR PRT (NYSE:F_pt) | $0.47 \%$ | Ford | 01.2003 |
| 2 | BARCLAYS BK PR | $0.58 \%$ | BCS (BARCLAYS PLC) | $-1.52 \%$ |
| 3 | GAB_P (GABELLI EQ PR) | $0.64 \%$ | GBL (GABELLI ASSET A) | $0.78 \%$ |
| 4 | DYNEX CAPTL PRB (NasdaqNM:DXCPO) | $2.05 \%$ | DYNEX CAPITAL (NYSE:DX) | $2.20 \%$ |
| 5 | J.P. MORGAN PR A (NYSE:JPM_pa) | $0.38 \%$ | JPM (JP MORGAN CHASE) | $1.11 \%$ |
| 6 | CAMECO CORP PR | $0.79 \%$ | CCJ (CAMECO CORP) | $0.68 \%$ |
| 7 | CCM_P (CARLTON COMM PR) | $0.66 \%$ | CCTVY (CARLTON COMM) | $1.43 \%$ |
| 8 | INTEGRA CAP PR | $0.36 \%$ | INTEGRA BANK CP (NasdaqNM:IBNK) | $-0.47 \%$ |
| 9 | MARINER CAP PR (NasdaqNM:FMARP) | $0.58 \%$ | FST MARINER (NasdaqNM:FMAR) | $2.52 \%$ |
| 10 | OI_PA (OWENS ILL PR A) | $1.17 \%$ | OI (OWENS-ILLINOIS) | $2.58 \%$ |
| 11 | NCX_P (NOVA CHEM CP PR) | $0.59 \%$ | NOVA CHEMICALS (NYSE:NCX) | $1.58 \%$ |
| 12 | ANZ_P (AUSTRALIA NZ PR) | $0.52 \%$ | ANZ BANKING GRP (NYSE:ANZ) | $1.57 \%$ |
| 13 | ROYCE VAL PR (NYSE:RVT_p) | $0.61 \%$ | RVT (ROYCE VALUE TR) | $0.90 \%$ |
| 14 | LKFNP (LAKELAND CAP PR) | $0.65 \%$ | LAKELAND FINL (NasdaqNM:LKFN) | $1.22 \%$ |
| 15 | IFC_PP (IFC CAP I PR P) | $0.68 \%$ | IFC (IRWIN FINL CORP) | $0.76 \%$ |
| 16 | SQA_P (SEQUA CORP PR) | $0.23 \%$ | SEQUA CORP A (NYSE:SQAa) | $-0.17 \%$ |
| 17 | ABANP (ABI CAP TR PR) | $0.73 \%$ | APPLIED BIOSYS (NYSE:ABI) | $1.47 \%$ |
| 18 | MER_PC (MERRILL PR C) | $0.64 \%$ | MER (MERRILL LYNCH) | $0.84 \%$ |
| 19 | N_PE (INCO PR E) | $1.53 \%$ | N (INCO LTD) | $2.56 \%$ |




Average return across firms


## THE COST OF PREFERRED STOCK

Preferred stock is a hybrid security that has features of both debt and equity. Preferred stock cannot be issued by $S$ corporations. In contrast, C corporations can issue preferred stock. In case of bankruptcy, preferred stockholders are paid before common stockholders, and therefore a firm's preferred stock is less risky than its common. The dividend on preferred stock represents an obligation of the corporation, and in this sense it is like interest payments on debt. While interest payments are a legal obligation of the firm, preferred dividends are akin to a moral obligation. If the firm does not pay the preferred dividend, the owner of the preferred stock cannot legally force the firm to pay it, and in this respect the preferred stock is like common equity. Typically, however, preferred dividends are cumulative. Preferred stock that is convertible to common stock is termed convertible preferred. The value of this preferred is equal to the value of a nonconvertible of equal risk plus the value of the conversion feature, which is a call option on the equity of the firm. Here, we value only a straight preferred. The cost of preferred equity is given by Equation 5.19.

$$
\begin{equation*}
V_{p s}=\frac{d i v_{p s}}{k_{p s}} \tag{5.19}
\end{equation*}
$$

Since $V_{p s}$ is not known for a private firm, $k_{p s}$ cannot be calculated from Equation 5.19. Therefore, we need to calculate $k_{p s}$ using another approach. Since preferred stock is less risky than common, $k_{p s}$ should be lower then $k_{e}$. This suggests that if we know the ratio of the average preferred stock return to the average common stock return then we can calculate $k_{e}$ using the buildup method and then multiply the result by the return ratio to estimate $k_{p s}$. Table 5.10 estimates the return ratio using a sample of 40 firms.

The data indicates that the preferred stock return on average is about 80 percent of the common stock return. Thus we can approximate the preferred stock return by multiplying the common stock return, estimated using the adjusted CAPM, by 80 percent. If the cost of equity is 25 percent, then the cost of a straight preferred can be approximated by $0.8 \times 25$ percent, or 20 percent.

## Calculating the Weighted Average Cost of Capital

Table 5.11 shows an example of estimating the weighted average cost of capital for a firm that has $\$ 10$ million in revenue.

The WACC is 25 percent. This rate is dominated by the cost of equity, because the capital structure assumed is 90 percent equity and 10 percent debt. As the debt percentage rises, the WACC will decline because the aftertax cost of debt is lower than the cost of equity. As noted in Chapter 2, as

TABLE 5.11 Weighted Average Cost of Capital for a $\$ 10$ Million Revenue Firm

| Row | Cost of Capital Components | Values | Source |
| :---: | :---: | :---: | :---: |
| 1 | Risk-free rate | 4.68\% | Text |
| 2 | Unlevered beta | 0.52 | Text |
| 3 | Beta adjustment factor for size and sum | 1.37 | Linear interpolation of values in Table 5.4 |
| 4 | Unlevered beta adjusted for size and sum | 0.71 | Calculated, text |
| 5 | Debt/equity ratio | 11.11\% | $90 \%$ equity, $10 \%$ debt: assumed |
| 6 | Tax rate | 0.4 | Statutory |
| 7 | Levered beta adjusted for size and sum | 0.76 | Calculated, equation $5.15$ |
| 8 | Risk premium | 7.42\% | Table 5.1 |
| 9 | Size premium | 8.91\% | Text and Table 5.5 |
| 10 | Firm-specific risk premium | 8.00\% | Text: Gompers and Learner |
| 11 | Cost of equity | 27.23\% | Calculated, Equation 5.2 |
| 12 | Debt cost | 8.21\% | Tentex example |
| 13 | Cost of preferred stock | 21.78\% | Text |
| 14 | Equity percentage | 90.00\% | Assumed |
| 15 | Debt percentage | 10.00\% | Assumed |
| 16 | Preferred stock percentage | 0.00\% | Assumed |
| 17 | WACC | 25.00\% | Calculated, Equation 5.1 |

more debt is used in the capital structure, the WACC will reach a minimum and then begin to rise. This occurs because at some point the additional risk created by the additional debt issued, measured as the increase in the present value of bankruptcy costs, is greater than the tax benefits from the incremental debt issuance.

## SUMMARY

This chapter addressed the issues in estimating the weighted average cost of capital and its components-the cost of equity, debt, and preferred stock. Using the buildup method, we estimated the cost of equity and proposed a method to make several adjustments to Ibbotson size premium to make it more useful in estimating the cost of equity for private firms. Altman's Z score model was used to estimate the base cost of debt for a private firm. To this value an increment was added based on firm size to obtain the final cost of debt. Finally, the cost of preferred stock was estimated by demonstrating that, on average, the preferred stock return is about 80 percent of the return on common equity.

